Analysis of Statically Determinate Structures

- Idealized Structure
- Principle of Superposition
- Equations of Equilibrium
- Determinacy and Stability
  - Beams
  - Frames
  - Gable Frames
- Application of the Equations of Equilibrium
- Analysis of Simple Diaphragm and Shear Wall Systems Problems
Classification of Structures

- Support Connections

  typical “pin-supported”
  connection (metal)

  typical “roller-supported”
  connection (concrete)

  typical “fixed-supported”
  connection (metal)

  typical “fixed-supported”
  connection (concrete)
fixed-connected joint

pin-connected joint

fixed support

pin support

fixed-connected joint

torsional spring support

torsional spring joint

P

A

B

L/2

L/2

actual beam

idealized beam
Table 2-1 Supports for Coplanar Structures

<table>
<thead>
<tr>
<th>Type of Connection</th>
<th>Idealized Symbol</th>
<th>Reaction</th>
<th>Number of Unknowns</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Light cable</td>
<td></td>
<td><img src="image" alt="Light Cable Reaction" /></td>
<td>One unknown. The reaction is a force that acts in the direction of the cable or link.</td>
</tr>
<tr>
<td>(2) rollers</td>
<td></td>
<td><img src="image" alt="Roller Reaction" /></td>
<td>One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.</td>
</tr>
<tr>
<td>rockers</td>
<td></td>
<td><img src="image" alt="Rockers Reaction" /></td>
<td></td>
</tr>
<tr>
<td>(3)</td>
<td></td>
<td><img src="image" alt="Rockers Reaction" /></td>
<td>One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.</td>
</tr>
<tr>
<td>(4)</td>
<td></td>
<td><img src="image" alt="Rollers Reaction" /></td>
<td>One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.</td>
</tr>
<tr>
<td>Type of Connection</td>
<td>Idealized Symbol</td>
<td>Reaction</td>
<td>Number of Unknowns</td>
</tr>
<tr>
<td>--------------------</td>
<td>------------------</td>
<td>----------</td>
<td>--------------------</td>
</tr>
<tr>
<td>Smooth pin or hinge</td>
<td><img src="image" alt="Smooth pin or hinge" /></td>
<td>$F_y$, $F_x$</td>
<td>Two unknowns. The reactions are two force components.</td>
</tr>
<tr>
<td>slider</td>
<td><img src="image" alt="Slider" /></td>
<td>$F$, $M$</td>
<td>Two unknowns. The reactions are a force and moment.</td>
</tr>
<tr>
<td>fixed-connected collar</td>
<td><img src="image" alt="Fixed-connected collar" /></td>
<td></td>
<td></td>
</tr>
<tr>
<td>fixed support</td>
<td><img src="image" alt="Fixed support" /></td>
<td>$F_x$, $M$, $F_y$</td>
<td>Three unknowns. The reactions are the moment and the two force components.</td>
</tr>
</tbody>
</table>
• Idealized Structure.
idealized framing plan

idealized framing plan
• Tributary Loadings.
One-Way System.
Idealized framing plan for one-way slab action requires \( \frac{L_2}{L_1} \geq 2 \)

Concrete slab is reinforced in two directions, poured on plane forms.
Two-Way System.

$L_2/L_1 = 1.0 < 2$

idealized framing plan

idealized beam, all
Two requirements must be imposed for the principle of superposition to apply:

1. The material must behave in a linear-elastic manner, so that Hooke’s law is valid, and therefore the load will be proportional to displacement.

\[ \sigma = \frac{P}{A} \]
\[ \delta = \frac{PL}{AE} \]

2. The geometry of the structure must not undergo significant change when the loads are applied, i.e., small displacement theory applies. Large displacements will significantly change and orientation of the loads. An example would be a cantilevered thin rod subjected to a force at its end.
Equations of Equilibrium

\[ \Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma F_z = 0 \]
\[ \Sigma M_x = 0 \quad \Sigma M_y = 0 \quad \Sigma M_z = 0 \]

internal loadings
Determinacy and Stability

- Determinacy

\[ r = 3n, \text{ statically determinate} \]
\[ r > 3n, \text{ statically indeterminate} \]

\( n \) = the total parts of structure members.
\( r \) = the total number of unknown reactive force and moment components
Example 2-1

Classify each of the beams shown below as statically determinate or statically indeterminate. If statically indeterminate, report the number of degrees of indeterminacy. The beams are subjected to external loadings that are assumed to be known and can act anywhere on the beams.
**SOLUTION**

1. \( r = 3, \ n = 1, \ 3 = 3(1) \)  
   - Statically **determinate**  

2. \( r = 5, \ n = 1, \ 5 - 3(1) = 2 \)  
   - Statically **indeterminate** to the second degree  

3. \( r = 6, \ n = 2, \ 6 = 3(2) \)  
   - Statically **determinate**  

4. \( r = 10, \ n = 3, \ 10 - 3(3) = 1 \)  
   - Statically **indeterminate** to the first degree
Example 2-2

Classify each of the pin-connected structures shown in figure below as statically determinate or statically indeterminate. If statically are subjected to arbitrary external loadings that are assumed to be known and can act anywhere on the structures.
SOLUTION

$r = 7, n = 2, 7 - 3(2) = 1$

Statically indeterminate to the first degree

$r = 9, n = 3, 9 = 3(3)$

Statically determinate
\[ r = 10, \ n = 2, \ 10 - 6 = 4 \]

Statically indeterminate to the fourth degree

\[ r = 9, \ n = 3, \ 9 = 3(3) \]

Statically determinate

\[ r = 10, \ n = 2, \ 10 - 6 = 4 \]

Statically indeterminate to the fourth degree

\[ r = 9, \ n = 3, \ 9 = 3(3) \]

Statically determinate
Example 2-3

Classify each of the frames shown in figure below as statically determinate or statically indeterminate. If statically indeterminate, report the number of degrees of indeterminacy. The frames are subjected to external loadings that are assumed to be known and can act anywhere on the frames.
SOLUTION

\[ r = 9, \; n = 2, \; 9 - 6 = 3 \]
Statically indeterminate to the third degree

\[ r = 15, \; n = 3, \; 15 - 9 = 6 \]
Statically indeterminate to the sixth degree
• Stability

\[ r < 3n, \text{ unstable} \]

\[ r \geq 3n, \text{ unstable if member reactions are concurrent or parallel or some of the components form a collapsible mechanism} \]

**Partial Constrains**
Improper Constraints
Example 2-4

Classify each of the structures in the figure below as stable or unstable. The structures are subjected to arbitrary external loads that are assumed to be known.
**SOLUTION**

The member is *stable* since the reactions are non-concurrent and nonparallel. It is also statically *determinate*.

The compound beam is *stable*. It is also *indeterminate* to the second degree.

The compound beam is *unstable* since the three reactions are all parallel.
The member is *unstable* since the three reactions are concurrent at $B$.

The structure is *unstable* since $r = 7$, $n = 3$, so that, $r < 3n$, $7 < 9$. Also, this can be seen by inspection, since $AB$ can move horizontally without restraint.
Application of the Equations of Equilibrium

\[ r = 9, \; n = 3, \; 9 = 3(3); \quad \text{statically determine} \]
\[ r = 6, \ n = 2, \ 6 = 3(2); \text{ statically determine} \]
Example 2-5

Determine the reactions on the beam shown.
SOLUTION

\[ \sum F_y = 0: \quad A_y - 229.5 + 179.69 = 0 \]
\[ A_y = 49.81 \text{ kN} \]

\[ \sum M_A = 0: \quad B_y(4) - (229.5)(3) + (132.5)(0.3) - 70 = 0 \]
\[ B_y = 179.69 \text{ kN} \]

\[ \sum F_x = 0: \quad A_x - 132.5 = 0 \]
\[ A_x = 132.5 \text{ kN} \]
Example 2-6

Determine the reactions on the beam shown.
SOLUTION

\[ A_x = 0 \]

\[ A_y = 60 - 60 = 0 \]

\[ A_y = 120 \text{ kN} \uparrow \]

\[ M_A - (60)(4) - (60)(6) = 0 \]

\[ M_A = 600 \text{ kN} \cdot \text{m} \]
Example 2-7

Determine the reactions on the beam shown. Assume $A$ is a pin and the support at $B$ is a roller (smooth surface).
**SOLUTION**

\[\Sigma M_A = 0: \quad -28(2) + N_B\sin 33.7(6) + N_B\cos 33.7(3) = 0\]

\[N_B = 9.61 \text{ kN}\]

\[\Sigma F_x = 0: \quad A_x - N_B\cos 33.7 = 0; \quad A_x = 9.61\cos 33.7 = 8 \text{ kN} \rightarrow\]

\[\Sigma F_y = 0: \quad A_y - 28 + 9.61\cos 33.7 = 0\]

\[A_y = 22.67 \text{ kN}, \uparrow\]
Example 2-8

The compound beam in figure below is fixed at $A$. Determine the reactions at $A$, $B$, and $C$. Assume that the connection at pin and $C$ is a rooler.
**Member BC**

\[ \sum M_B = 0: \quad C_y(4) - 8 = 0 \]
\[ C_y = 2 \text{ kN} \uparrow \]

\[ \sum F_x = 0: \quad B_x = 0 \]

\[ \sum F_y = 0: \quad C_y - B_y = 0; \quad B_y = C_y = 2 \text{ kN} \uparrow \]

**Member AB**

\[ \sum M_A = 0: \quad M_A - 36(3) + 2(6) = 0 \]
\[ M_A = 96 \text{ kN} \cdot \text{m} \]

\[ \sum F_x = 0: \quad A_x - B = 0; \quad A_x = B_x = 0 \]

\[ \sum F_y = 0: \quad A_y - 36 + 2 = 0 \]
\[ A_y = 34 \text{ kN} \uparrow 38 \]
Example 2-9

The side girder shown in the photo supports the boat and deck. An idealized model of this girder is shown in the figure below, where it can be assumed $A$ is a roller and $B$ is a pin. Using a local code the anticipated deck loading transmitted to the girder is 6 kN/m. Wind exerts a resultant horizontal force of 4 kN as shown, and the mass of the boat that is supported by the girder is 23 Mg. The boat’s mass center is at $G$. Determine the reactions at the supports.
**SOLUTION**

\[ \Sigma F_x = 0: \]
\[ 4 - B_x = 0 \]
\[ B_x = 4 \text{kN} , \leftarrow \]

\[ \Sigma M_B = 0: \]
\[ 22.8(1.9) - A_y(2) + 225.6(5.4) - 4(0.3) = 0 \]
\[ A_y = 630.2 \text{kN} , \uparrow \]

\[ \Sigma F_y = 0: \]
\[ -225.6 + 630.2 - 22.8 + B_y = 0 \]
\[ B_y = 382 \text{kN} , \uparrow \]
Example 2-10

Determine the horizontal and vertical components of reaction at the pins $A$, $B$, and $C$ of the two-member frame shown in the figure below.
**SOLUTION**

Member \( BC \)

\[ \Sigma M_C = 0: \]
\[ -B_y(2) + 6(1) = 0 \]
\[ B_y = 3 \text{ kN} , \uparrow \]

Member \( AB \)

\[ \Sigma M_A = 0: \]
\[ -8(2) - 3(2) + B_x(1.5) = 0 \]
\[ B_x = 14.7 \text{ kN} , \leftarrow \]

\[ \Sigma F_x = 0: \]
\[ A_x + (3/5)8 - 14.7 = 0 \]
\[ A_x = 9.87 \text{ kN} , \rightarrow \]

\[ \Sigma F_y = 0: \]
\[ A_y - (4/5)8 - 3 = 0 \]
\[ A_y = 9.4 \text{ kN} , \uparrow \]

Member \( BC \)

\[ \Sigma F_x = 0: \]
\[ C_x - B_x = 0; C_x = B_x = 14.7 \text{ kN} , \leftarrow \]

\[ \Sigma F_y = 0: \]
\[ 3 - 6 + C_y = 0 ; \quad C_y = 3 \text{ kN} , \uparrow \]
Example 2-11-1

From the figure below, determine the horizontal and vertical components of reaction at the pin connections $A$, $B$, and $C$ of the supporting gable arch.
SOLUTION

 Entire Frame

\[ \sum M_A = 0: \quad C_y (6) - 15(3) = 0 \]

\[ C_y = 7.5 \text{ kN} , \uparrow \]

\[ \sum F_y = 0: \quad A_y + 7.5 = 0 \]

\[ A_y = -7.5 \text{ kN} , \downarrow \]
Member $AB$

\[ \sum M_B = 0: \quad 15(3) + A_x(6) + 7.5(3) = 0 \]
\[ A_x = -11.25 \text{ kN} \]

\[ \sum F_x = 0: \quad -11.25 + 15 - B_x = 0 \]
\[ B_x = 3.75 \text{ kN} \]

\[ \sum F_y = 0: \quad -7.5 + B_y = 0 \]
\[ B_y = 7.5 \text{ kN} \]

Member $BC$

\[ \sum F_x = 0: \quad 3.75 - C_x = 0 \]
\[ C_x = 3.75 \text{ kN} \]
Example 2-11-2

The side of the building in the figure below is subjected to a wind loading that creates a uniform *normal* pressure of 1.5 kPa on the windward side and a suction pressure of 0.5 kPa on the leeward side. Determine the horizontal and vertical components of reaction at the pin connections *A*, *B*, and *C* of the supporting gable arch.
A uniform distributed load on the *windward* side is

\[(1.5 \text{ kN/m}^2)(4 \text{ m}) = 6 \text{ kN/m}\]

A uniform distributed load on the *leeward* side is

\[(0.5 \text{ kN/m}^2)(4 \text{ m}) = 2 \text{ kN/m}\]
Entire Frame

\[ \sum M_A = 0: \quad -(18+6)(1.5) - (25.46+8.49)\cos 45^\circ(4.5) - (25.46 \sin 45^\circ)(1.5) \]
\[ + (8.49 \sin 45^\circ)(4.5) + C_y(6) = 0 \]
\[ C_y = 24.0 \text{ kN} \]

\[ \sum F_y = 0: \quad A_y - 25.46 \sin 45^\circ + 8.49 \sin 45^\circ 3 + 24 = 0 \]
\[ A_y = -12.0 \text{ kN} \]
Member $AB$

\[ \sum M_B = 0: \quad (25.46 \sin 45^\circ)(1.5) + (25.46 \cos 45^\circ)(1.5) + (18)(4.5) + A_x(6) + 12(3) = 0 \]

\[ A_x = -28.5 \text{ kN} \quad \leftarrow \]

\[ \Sigma F_x = 0: \quad -28.5 + 18 + 25.46 \cos 45^\circ - B_x = 0 \]

\[ B_x = 7.5 \text{ kN} \quad \leftarrow \]

\[ \Sigma F_y = 0: \quad -12 - 25.46 \sin 45^\circ + B_y = 0 \]

\[ B_y = 30.0 \text{ kN} \uparrow \]

Member $CB$

\[ \Sigma F_x = 0: \quad 7.5 + 8.49 \cos 45^\circ + 6 - C_x = 0 \]

\[ C_x = 19.50 \text{ kN} \quad \leftarrow \]
Analysis of Simple Diaphragm and shear Wall Systems
Example 2-12

Assume the wind loading acting on one side of a two-story building is as shown in the figure below. If shear walls are located at each of the corners as shown and flanked by columns, determine the shear in each panel located between the floors and the shear along the columns.
**SOLUTION**

\[
F_{R1} = 0.8 \times 10^3 \text{ N/m}^2 \times (20 \text{ m}) \times (4 \text{ m}) = 64 \text{ kN}
\]

\[
F_{R1}/2 = 32
\]

\[
F_{R2} = 1.2 \times 10^3 \text{ N/m}^2 \times (20 \text{ m}) \times (4 \text{ m}) = 96 \text{ kN}
\]

\[
F_{R2}/2 = 48
\]
\[ \Sigma M = 0: \]
\[ F_v (3) - 12(4) = 0 \]
\[ F_v = 16 \text{ kN} \]

\[ \Sigma M = 0: \]
\[ F'_v (3) - 32(4) = 0 \]
\[ F'_v = 42.7 \text{ kN} \]