Subjects of interest

• Objectives / Introduction
• Stress cycles
• The S-N curve
• Cyclic stress-strain curve
• Low cycle fatigue
• Structural features of fatigue
• Fatigue crack propagation
• Factors influencing fatigue properties
• Design for fatigue
Objectives

• This chapter provides fundamental aspects of fatigue in metals and the significance of fatigue failure.

• Different approaches for the assessment of fatigue properties, i.e., fatigue S-N curve and fatigue crack growth resistance will be introduced.

• Discussion will be made on factors influencing fatigue properties of metals, for example, mean stress, stress concentration, temperature

• Finally design against fatigue failure will be highlighted.
Introduction

Fatigue failure in a bolt

Fatigue initiation

Beach mark
Introduction

Fatigue failure occurs at the outer rim of the wheel

Fatigue fracture area in a shaft caused by corroded inside area
Introduction

**Fatigue failures are widely studies because it accounts for 90% of all service failures due to mechanical causes.**

*Characteristics*

• Fatigue failures occur when metal is subjected to a **repetitive or fluctuating stress** and will fail at a **stress much lower than its tensile strength**.

• Fatigue failures occur without any **plastic deformation** (no warning).

• Fatigue surface appears as a smooth region, showing **beach mark** or origin of fatigue crack.
Factors causing fatigue failure

**Basic factors**

1) A maximum tensile stress of sufficiently high value.
2) A large amount of variation or fluctuation in the applied stress.
3) A sufficiently large number of cycles of the applied stress.

**Additional factors**

- Stress concentration
- Corrosion
- Temperature
- Overload
- Metallurgical structure
- Residual stress
- Combined stress
**Stress cycles**

**Different types of fluctuating stress**

(a) Completely reversed cycle of stress (sinusoidal)

(b) Repeated stress cycle

(c) Irregular or random stress cycle

\[
\sigma_{\text{max}} = - \sigma_{\text{min}}
\]

Tensile stress +

Compressive stress -
Stress cycles

Nomenclature of stress parameter in fatigue loading

Maximum stress, $\sigma_{\text{max}}$

Minimum stress, $\sigma_{\text{min}}$

Stress range

\[ \Delta \sigma \quad \text{or} \quad \sigma_r = \sigma_{\text{max}} - \sigma_{\text{min}} \]  \hspace{1cm} \text{Eq.1}

Alternating stress

\[ \sigma_a = \frac{\Delta \sigma}{2} = \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2} \]  \hspace{1cm} \text{Eq.2}

Mean stress

\[ \sigma_m = \frac{\sigma_{\text{max}} + \sigma_{\text{min}}}{2} \]  \hspace{1cm} \text{Eq.3}

Stress ratio

\[ R = \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}} \]  \hspace{1cm} \text{Eq.4}

Amplitude ratio

\[ A = \frac{\sigma_a}{\sigma_m} = \frac{1-R}{1+R} \]  \hspace{1cm} \text{Eq.5}

Fatigue stress cycle
The S-N curve

• **Engineering fatigue data** is normally represented by means of the **S-N curve**, a plot of stress $S$ against the number of cycle, $N$.

• Stress can be $\sigma_a$, $\sigma_{\text{max}}$, $\sigma_{\text{min}}$.

• $\sigma_{\text{m}}$, $R$ or $A$ should be mentioned.

• **S-N curve** is concerned chiefly with **fatigue failure at high numbers of cycles** ($N > 10^5$ cycles) $\Rightarrow$ high cycle fatigue (HCF).

• $N < 10^4$ or $10^5$ cycles $\Rightarrow$ low cycle fatigue (LCF).

• $N$ increases with decreasing stress level.

  • **Fatigue limit or endurance limit** is normally defined at $10^7$ or $10^8$ cycles. Below this limit, the material presumably can endure an infinite number of cycle before failure.

  • **Nonferrous metal**, i.e., aluminium, do not have fatigue limit $\Rightarrow$ fatigue strength is defined at $\sim 10^8$ cycles.
The **S-N curve** in the **high-cycle region** is sometimes described by the **Basquin equation**

\[ N\sigma_a^p = C \]  

**Eq.6**

Where \( \sigma_a \) is the stress amplitude, \( p \) and \( C \) are empirical constants.

**HCF**  
High cycle (low strain) fatigue

**LCF**  
Low cycle (high strain) fatigue
Construction of S-N curve

- The construction of S-N curve normally requires ~ 8-12 specimens by first testing at a high level of stress ~ 2/3 of the tensile strength of the material.

- The test is then carried out at lower levels of stress until runout.

- The data obtained is normally scattered at the same stress level by using several specimens.

- This requires statistic approach to define the fatigue limit.
Statistical nature of fatigue

- Because the **S-N fatigue data** is normally scattered, it should be therefore represented on a **probability basis**.
- Considerable number of specimens are used to obtain statistical parameters.
- At $\sigma_1$, 1% of specimens would be expected to fail at $N_1$ cycles.
- 50% of specimens would be expected to fail at $N_2$ cycles.

Note: The S-N fatigue data is more scattered at lower stress levels. Each specimen has its own fatigue limit.

• For **engineering purposes**, it is sufficiently accurate to assume a **logarithmic normal distribution of fatigue life** in the region of the probability of failure of $P = 0.10$ to $P = 0.90$. 

Fatigue data on a probability basis
Effect of mean stress, stress range and stress intensity (notch) on S-N fatigue curve

\[ K_t = \frac{\sigma_{loc}}{\sigma_{app}} \]

- Mean stress
- Stress range
- Stress intensity

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**Goodman diagram**

- **Goodman diagram** shows the variation of the limiting range of stress ($\sigma_{\text{max}} - \sigma_{\text{min}}$) on mean stress.
- As the *mean stress* becomes more tensile the *allowable range of stress* is reduced.
- At tensile strength, $\sigma_u$, the stress range is zero.
In the Haig-Solderberg diagram, a plot of alternating stress $\sigma_a$ and mean stress $\sigma_m$ is used. The Goodman relationship may be expressed by:

$$\sigma_a = \sigma_e \left[ 1 - \left( \frac{\sigma_m}{\sigma_u} \right)^x \right]$$

Eq. 7

Where $x = 1$ for the Goodman line, $x = 2$ for the Gerber parabola, $\sigma_e$ is the fatigue limit for completely reversed loading.

If the design is based on the yield strength $\sigma_o$, (based on the Solderberg line), then the $\sigma_u$ is replaced by $\sigma_o$ in this equation.
Master diagram for establishing influence of mean stress in fatigue

Ex: \(\sigma_{\text{max}} = 400\ \text{MPa, } \sigma_{\text{min}} = 0\), a fatigue limit of the notched specimen is less than \(10^6\) cycles.

For the unnotched specimen is below the fatigue limit.
**Example:** A 4340 steel bar is subjected to a fluctuating axial load that varies from a maximum of 330 kN tension to a minimum of 110 kN compression. The mechanical properties of the steel are:

\[ \sigma_u = 1090 \text{ MPa}, \quad \sigma_o = 1010 \text{ MPa}, \quad \sigma_e = 510 \text{ MPa} \]

Determine the bar diameter to give infinite fatigue life based on a safety factor of 2.5.

Cylindrical cross section of the bar = \( A \), the variation of stress will be

\[
\sigma_{max} = \frac{0.330}{A} \text{ MPa}, \quad \sigma_{min} = -\frac{0.110}{A} \text{ MPa}
\]

\[
\sigma_{mean} = \frac{\sigma_{max} + \sigma_{min}}{2} = \frac{0.330}{A} + \left(-\frac{0.110}{A}\right) = \frac{0.110}{A} \text{ MPa}
\]

\[
\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{0.330}{A} - \left(-\frac{0.110}{A}\right) = \frac{0.220}{A} \text{ MPa}
\]

Using the **conservative Goodman line** and Eq. 7.

\[
\sigma_a = \sigma_e \left(1 - \frac{\sigma_m}{\sigma_u}\right), \quad \sigma_e = \frac{510}{2.5} = 204 \text{ MPa}
\]

\[
\frac{0.220}{A} = 1 - \frac{0.110}{1090}
\]

\[
A = 1179 \text{ mm}^2
\]

\[
D = \sqrt{\frac{4A}{\pi}} = 38.7 \text{ mm}
\]
Cyclic stress-strain curve

- Cyclic strain controlled fatigue occurs when the strain amplitude is held constant during cycling.
- Found in thermal cycling where a component expands and contracts in response to fluctuations in the operating temperature or in reversed bending between fixed displacements.

During the initial loading, the stress-strain curve is **O-A-B**.

Yielding begins on unloading in compression at a lower stress **C** due to the **Bauschinger effect**.

A hysteresis loop develops in reloading with its dimensions of width, **Δε** and height **Δσ**.

The total strain range **Δε** consists of the elastic strain component plus the plastic strain component.

\[ Δε = Δε_e + Δε_p \]  

Eq.8
Cyclic hardening and cyclic softening

- **Cyclic hardening** would lead to a decreasing peak strain with increasing cycles. \(n > 0.15\)

- **Cyclic softening** would lead to a continually increasing strain range and early fracture. \(n < 0.15\)
Comparison of monotonic and cyclic stress-strain curves of cyclic hardened materials

- The cycle stress-strain curve may be described by a power curve as follows:

$$\Delta \sigma = K' \left( \Delta \varepsilon_p \right)^{n'}$$  \hspace{1cm} \text{Eq.9}

Where $n'$ is the cyclic strain-hardening exponent.

- $K'$ is the cyclic strength coefficient.

Since strain amplitude

$$\frac{\Delta \varepsilon}{2} = \frac{\Delta \varepsilon_e}{2} + \frac{\Delta \varepsilon_p}{2}$$

$$\frac{\Delta \varepsilon}{2} = \frac{\Delta \sigma}{2E} + \frac{1}{2} \left( \frac{\Delta \sigma}{K'} \right)^{1/n'}$$

For metals $n'$ varies between 0.10 - 0.20.
Low cycle fatigue

- **Low cycle fatigue** (LCF) (high strain) is concerned about fatigue failure at relatively **high stress** and **low numbers of cycles** to failure.
- **Ex:** in the nuclear pressure vessels, steam turbines and power machinery. Usually concerned with **cyclic strain** rather than **cyclic stress**.
- **LCF** data is normally present as a plot of strain range \( \Delta \varepsilon_p \) against \( N \).

On the log scale, this relation can be best described by

\[
\frac{\Delta \varepsilon_p}{2} = \varepsilon_f' (2N)^c
\]

Eq.10

Where
\( \Delta \varepsilon_p/2 \) = plastic strain amplitude
\( \varepsilon_f' \) = fatigue ductility coefficient
\( 2N \) = number of strain reversals to failure.
\( c \) = fatigue ductility exponent varies between -0.5 to -0.7.
Example: For the cyclic stress-strain curve, $\sigma_B = 75$ MPa and $\varepsilon_B = 0.000645$. If $\varepsilon_f = 0.30$ and $E = 22 \times 10^4$ MPa.

Determine

(a) $\Delta \varepsilon_e$ and $\Delta \varepsilon_p$

\[
\Delta \varepsilon_e = \frac{\Delta \sigma}{E} = \frac{2(75)}{22 \times 10^4} = 6.818 \times 10^{-4}
\]

\[
\Delta \varepsilon_p = \Delta \varepsilon - \Delta \varepsilon_e = (2 \times 0.000645) - 0.0006818 = 6.082 \times 10^{-4}
\]

(b) The number of cycles to failure.

From the Coffin-Manson relation

\[
\frac{\Delta \varepsilon_p}{2} = \varepsilon'_f (2N)^c
\]

If $c = -0.6$ and $\varepsilon'_f \sim \varepsilon'_f$

\[
\frac{6.082 \times 10^{-4}}{2} = 0.30(2N)^{-0.6}
\]

\[
N = 49,000 \text{ cycles}
\]
For the high-cycle (low strain) fatigue (HCF) regime, where the nominal strains are elastic, Basquin’s equation can be reformulated to give

\[ \frac{\Delta \varepsilon}{2} = \frac{\Delta \varepsilon_e}{2} + \frac{\Delta \varepsilon_p}{2} \]

\[ \frac{\Delta \varepsilon}{2} = \frac{\sigma'_f}{E} (2N)^b + \varepsilon'_f (2N)^c \]

Where
- \( \sigma_a \) = alternate stress amplitude
- \( \Delta \varepsilon_e/2 \) = elastic strain amplitude
- \( E \) = Young’s modulus
- \( \sigma'_f \) = fatigue strength coefficient defined by the stress intercept at 2N=1.
- \( 2N \) = number of load reversals to failure (N = number of cycles to failure)
- \( b \) = fatigue strength exponent, which varies between –0.05 and -0.12 for most metals.
The fatigue strain-life curve

- tends toward the plastic curve at large total strain amplitudes
- tends toward the elastic curve at small total strain amplitudes.

**Ductile materials**
- High cyclic strain condition

**Strong materials**
- Low cyclic strain condition

The fatigue life value at which this transition occurs is

\[ 2N_t = \left( \frac{\varepsilon_f \cdot E}{\sigma_f} \right)^{1/(b-c)} \]  

Eq. 12

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Structural features of fatigue

The fatigue process can be divided into the following processes:

1) **Crack initiation**: the early development of fatigue damage (can be removed by a suitable thermal anneal).

2) **Slip band crack growth**: the deepening of the initial crack on plane of high shear stress (stage I crack growth)

3) **Crack growth on planes of high tensile stress**: growth of well-defined crack in direction normal to maximum tensile stress

4) **Ultimate ductile failure**: occurs when the crack reaches sufficient length so that the remaining cross section cannot support the applied load.
Initiation of fatigue crack and slip band crack growth (stage I)

- **Fatigue cracks** are normally initiated at a **free surface**. **Slip lines** are formed during the first few thousand cycles of stress.

- Back and forth fine slip movements of fatigue could build up notches or ridges at the surface. → act as **stress raiser** → initiate **crack**.

![Diagram showing fatigue crack growth stages]

- In **stage I**, the fatigue crack tends to propagate initially along slip planes (**extrusion** and **intrusion** of **persistent slip bands**) and later take the direction normal to the maximum tensile stress (**stage II**).

- The crack propagation rate in **stage I** is generally very low on the order of nm/cycles → giving **featureless surface**.
**Stable crack growth (stage II)**

- The fracture surface of stage II crack propagation frequently shows a pattern of ripples or **fatigue striations**.
- Each **striation** is produced by a **single stress cycle** and represents the successive position of an advancing crack front normal to the greatest tensile stress.

**Plastic blunting model of fatigue striation**

- **Crack tip blunting** occurs during **tensile load** at 45° and crack grows longer by **plastic shearing**.
- **Compression load** reverses the slip direction in the end zones → crushing the crack surface to form a **resharpened crack tip**.

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Fatigue crack propagation

Stage I
Non-propagating fatigue crack (~0.25nm/cycle)

Stage II
Stable fatigue crack propagation- widely study

Stage III
Unstable fatigue crack propagation \( \rightarrow \) failure

- For design against fatigue failure, fracture mechanics is utilised to monitor the fatigue crack growth rate in the stage II Paris regime.

\[
\frac{da}{dN} = A(\Delta K)^m
\]

Eq.13

- Where the fatigue crack growth rate \( \frac{da}{dN} \) varies with stress intensity factor range \( \Delta K \), which is a function of stress range \( \Delta \sigma \) and crack length \( a \).

\[
\Delta K = K_{max} - K_{min}
\]

\[
\Delta K = \sigma_{max} \sqrt{\pi a} - \sigma_{min} \sqrt{\pi a}
\]

Eq.14

A log scale plot gives Paris exponent \( m \) as the slope

FCG curve
# Fatigue Crack Propagation

**Stage I**  
Non-propagating fatigue crack (~0.25 nm/cycle)

**Stage II**  
Stable fatigue crack propagation - widely study

**Stage III**  
Unstable fatigue crack propagation → failure

## Fatigue Crack Growth Behaviour

<table>
<thead>
<tr>
<th>Non-continuum behaviour</th>
<th>Continuum behaviour (striations) or transition from non-continuum behaviour with Small to large influence of microstructure, depending on the material</th>
<th>Static mode of behaviour (cleavage, intergranular and dimples)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large influence of • microstructure • mean stress • environment</td>
<td>Large influence of • certain combination of environment, mean stress and frequency</td>
<td>Large influence of • microstructure • mean stress • thickness</td>
</tr>
<tr>
<td>Little influence of • environment</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Fatigue Crack Growth Rate

\[
\frac{da}{dN} = a(\Delta K)^m
\]

- \( \Delta K_{th} \) for linear portion
- \( a(m) \) for unstable crack growth

### Stress Intensity Factor Range, \( \Delta K \) (log scale)
Fatigue crack growth propagation in stage II regime

Stage II fatigue crack growth propagation has been widely investigated in order to determine the fatigue crack growth life from the representing stable fatigue crack growth rate.

\[ \frac{da}{dN} = A(\Delta K)^m \]

\[ da/dN \]

The fatigue crack growth life \( N_f \) (stage II) can be determined by

\[ N_f = \int_0^{N_f} dN \]

\[ N_f = \frac{a_f^{-(m/2)+1} - a_i^{-(m/2)+1}}{(-m/2 + 1) A \sigma^m r^{m/2} \alpha^m} \]

where \( m \neq 2 \)

\( \alpha \) is the crack geometry factor

Fatigue crack growth in base metal and welded materials

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Example: A mild steel plate is subjected to constant amplitude uniaxial fatigue loads to produce stresses varying from $\sigma_{\text{max}} = 180 \text{ MPa}$ to $\sigma_{\text{min}} = -40 \text{ MPa}$. The static properties of the steel are $\sigma_o = 500 \text{ MPa}$, $\sigma_u = 600 \text{ MPa}$, $E = 207 \text{ MPa}$, and $K_c = 100 \text{ MPa.m}^{1/2}$. If the plate contains an initial through thickness edge crack of 0.5 mm, how many fatigue cycles will be required to break the plate?

For through thickness edge crack, $\alpha = 1.12$, and for ferritic-pearlitic steels, $A = 6.9 \times 10^{-12} \text{ MPam}^{1/2}$ and $m = 3.0$.

$\sigma_r = (180-0)$, since compressive stress are ignored, and neglect the influence of mean stress on the crack growth.

\[
a_i = 0.0005 \text{ m}, \quad a_f = \frac{1}{\pi} \left( \frac{K_c}{\sigma_{\text{max}} \alpha} \right)^2 = \frac{1}{\pi} \left( \frac{100}{180 \times 1.12} \right)^2 = 0.078 \text{ m}
\]

From Eq.15

\[
N_f = \frac{a_f^{-(m/2)+1} - a_i^{-(m/2)+1}}{(-(m/2)+1)A \sigma_r^m \pi^{m/2} \alpha^m}
\]

\[
N_f = \frac{(0.078)^{-(3/2)+1} - (0.0005)^{-(3/2)+1}}{(-(3/2)+1)(6.9 \times 10^{-12})(180)^3 (\pi)^{3/2}(1.12)^3} = 261,000 \text{ cycles}
\]
**S-N curve fracture surfaces**

- **S-N curve test** involves crack initiation and crack propagation to failure. → overall fatigue life.

- Fatigue testing normally uses *plain specimens* of different specimen surface conditions, i.e., polished, ground, machined, etc. under tension or bending.

- Crack initiation might be due to inclusions, second phases, porosity, defects.

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**Fatigue crack initiation from porosity**

**Fatigue crack initiation from inclusion/particle.**

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• **Fatigue crack growth (FCG)** test involves only **crack propagation** stage but excludes **crack initiation** stage.

• Specimen has an **initial short or small crack** and this crack will propagate under cyclic loading.

\[
\frac{da}{dN} = a(\Delta K)^m
\]

**Stage I**: Brittle facet (slip) + fatigue striation

**Stage II**: Fatigue striation

**Stage III**: Brittle facets (cleavage) + microvoids

**Stress intensity factor range, \(\Delta K\) (log scale)**
Factors influencing fatigue properties

- Stress concentration
- Size effect
- Surface effects
- Combined stresses
- Cumulative fatigue damage and sequence effects
- Metallurgical variables
- Corrosion
- Temperature
Effect of stress concentration on fatigue

Should avoid stress raisers from machining and fabrication processes.

- The effect of stress raiser or notch on fatigue strength can be determined by comparing the S-N curve of notched and unnotched specimens. (based on the net section of specimen).

- The notch sensitivity factor $q$ in fatigue is determined from

$$ q = \frac{K_f - 1}{K_t - 1} \quad \text{Eq.16} $$

Where

- $K_t$ is theoretical stress-concentration factor, depending on elasticity of crack tip
- $K_f$ is fatigue notch factor, ratio of fatigue strength of notched and unnotched specimens.

S-N curve of notched and unnotched specimens
# Size effect on fatigue

<table>
<thead>
<tr>
<th>Fatigue properties</th>
<th>Experimental scale</th>
<th>Industrial scale</th>
<th>Due to size effect</th>
</tr>
</thead>
</table>

- Fatigue property is **better** in the small sized specimens.
- Problem: the machine cannot accommodate *large specimens*.
- **Larger specimens** → increases *surface area* subjected to cyclic load, higher possibility to find defects on surface.
  - decreases the stress gradient and increases the *volume of material* which is highly stressed.

**Explain:** It is usually impossible to duplicate the same *stress concentration* and *stress gradient* in a small-sized laboratory specimens.

**Solution:** Use statistic approaches, i.e., *Weibull statistics*.
Surface effects on fatigue

- **Fatigue properties** are very sensitive to **surface conditions**.
- **Fatigue initiation** normally starts at the surface since the **maximum stress** is at the surface.

The factors which affect the surface of a fatigue specimen can be roughly divided into **three categories**;

- **Surface roughness**
- **Changes in surface properties**
- **Surface residual stress**
Surface roughness

- Different surface finishes produced by different machining processes can appreciably affect fatigue performance.

- Polished surface (very fine scratches), normally known as ‘par bar’ which is used in laboratory, gives the best fatigue strength.

<table>
<thead>
<tr>
<th>Type of finish</th>
<th>Surface roughness, μm</th>
<th>Median fatigue life, cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lathe-formed</td>
<td>2.67</td>
<td>24,000</td>
</tr>
<tr>
<td>Partly hand-polished</td>
<td>0.15</td>
<td>91,000</td>
</tr>
<tr>
<td>Hand-polished</td>
<td>0.13</td>
<td>137,000</td>
</tr>
<tr>
<td>Ground</td>
<td>0.18</td>
<td>217,000</td>
</tr>
<tr>
<td>Ground and polished</td>
<td>0.05</td>
<td>234,000</td>
</tr>
<tr>
<td>Superfinished</td>
<td>0.18</td>
<td>212,000</td>
</tr>
</tbody>
</table>


Reduction factor for fatigue limit of steel due to various surface treatments
Changes in surface properties

- Changes in surface properties due to surface treatments

**Treatments which reduces fatigue performance**

- **Decarburization**  
  *Ex:* decarburization of surface of heat-treated steels.

- **Soft coating**  
  *Ex:* Soft aluminium coating on an age-hardenable Al alloy.

- **Electroplating**
  Might reduces fatigue strength due to changes in residual stress, adhesion, porosity, hardness.

**Treatments which improves fatigue performance**

- **Carburizing**
- **Nitriding**
- **Flame hardening**
- **Induction hardening**

  - Forming harder and stronger surface → introducing compressive residual stress.
  - The strengthening effect depends on the diameter of the part and the depth of the surface hardening.
Surface residual stress

- Residual stresses arise when plastic deformation is not uniform throughout the entire cross section of the part being deformed.

<table>
<thead>
<tr>
<th>Loading</th>
<th>Unloading</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part undergone plastically deformed in tension</td>
<td>Compressive residual stress</td>
</tr>
<tr>
<td>Part undergone plastically deformed in compression</td>
<td>Tensile residual stress</td>
</tr>
</tbody>
</table>

(a) Shows the elastic stress distribution in a beam with no residual stress.

(b) Typical residual stress distribution produced by shot peening where the high compressive stress is balanced by the tensile stress underneath.

(c) The stress distribution due to the algebraic summation of the external bending stress and the residual stress.
Commercial methods introducing favourable compressive stress

- **Surface rolling**
  - Compressive stress is introduced in between the rollers during sheet rolling.

- **Shot peening**
  - Projecting fine steel or cast-iron shot against the surface at high velocity.

- **Polishing**
  - Reducing surface scratches

- **Thermal stress**
  - Quenching or surface treatments introduce volume change → giving compressive stress.
Effect of combined stresses on fatigue

Few data has been made on fatigue test with different combinations of types of stresses.

• **Ductile metals** under combined bending and torsion fatigue follow a distortion-energy (von Mises).

• **Brittle materials** follows the maximum principal stress theory (Tresca).

*Sines* has proposed expressions for

\[
\begin{align*}
\text{Low strain} & : \quad \left[ (\sigma_{a1} - \sigma_{a2})^2 + (\sigma_{a2} - \sigma_{a3})^2 + (\sigma_{a3} - \sigma_{a1})^2 \right]^{1/2} + C_2 (m_1 + m_2 + m_3) \geq \frac{\sqrt{2} \sigma_a}{K_f} \\
\text{High strain} & : \quad \varepsilon_q = \frac{\gamma_{oct}}{2} = \frac{1}{3} \left[ (\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2 \right]^{1/2}
\end{align*}
\]

**Note:** Effects of residual stress and triaxial stress are included.
Cumulative fatigue damage and sequence effects on fatigue

Practically, **levels of stress** are not held constant as in **S-N tests**, but can vary below or above the designed stress level.

- **Overstressing**: The initial applied stress level is higher than the fatigue limit for a short period of time beyond failure, then cyclic stressing below the fatigue limit. This overstressing **reduces the fatigue limit**.

- **Understressing**: The initial applied stress level is lower than the fatigue limit for a period of time, then cyclic stressing above the fatigue limit. This understressing **increases the fatigue limit** (might be due to strain hardening on the surface).
Cumulative damage rule

The percentage of fatigue life consumed by operation at one operating stress level depends on the magnitude of subsequent stress levels → the cumulative rule called **Miner’s rule**.

\[
\frac{n_1}{N_1} + \frac{n_2}{N_2} + \ldots + \frac{n_k}{N_k} = 1 \quad \text{or} \quad \sum_{j=1}^{k} \frac{n_j}{N_j}
\]

Eq. 19

Where \( n_1, n_2, \ldots n_k \) = the number of cycles of operation at specific over-stress levels. 
\( N_1, N_2, \ldots N_k \) = the life (in cycles) at this same over-stress level.

**Note:** for notched specimen, the fatigue strength is reduced much more than it would be predicted from the **Miner’s linear damage rule**. This is due to the effect of residual stress produced at the notch by overload stresses in the plastic region.
**Example:** A plain sided specimen is subjected to $1 \times 10^7$ cycles, at an applied stress range of 200 MPa. Estimate how many further cycles can be applied at a stress range of 500 MPa before failure is predicted to occur.

### Given information

<table>
<thead>
<tr>
<th>Applied stress range (MPa)</th>
<th>Number of cycles to failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>600</td>
<td>$1 \times 10^4$</td>
</tr>
<tr>
<td>500</td>
<td>$2 \times 10^4$</td>
</tr>
<tr>
<td>400</td>
<td>$5 \times 10^4$</td>
</tr>
<tr>
<td>300</td>
<td>$3 \times 10^5$</td>
</tr>
<tr>
<td>250</td>
<td>$3 \times 10^6$</td>
</tr>
<tr>
<td>200</td>
<td>$8 \times 10^7$</td>
</tr>
</tbody>
</table>

From Miner’s rule

**Assumption:** the total life of a part can be estimated by adding up the percentage of life consumed by each overstress cycle.

Therefore, the specimen can further withstand the fatigue load at 500 MPa for

\[
\frac{n_1}{N_1} + \frac{n_2}{N_2} + \ldots + \frac{n_k}{N_k} = 1
\]

\[
\frac{1 \times 10^7}{8 \times 10^7} + \frac{x}{2 \times 10^4} = 1
\]

\[
x = \frac{7}{8} \times 2 \times 10^4 = 1.75 \times 10^4 \text{ cycles}
\]
Effects of metallurgical variables on fatigue

- Fatigue property is normally greatly improved by changing the designs or, reducing stress concentration, introducing compressive stress on the surface.

- Few attempts have paid on improving metallurgical structure to improve fatigue properties but it is still important.

- Fatigue property is frequently correlated with tensile properties.

<table>
<thead>
<tr>
<th>Fatigue ratio =</th>
<th>Fatigue strength</th>
<th>Tensile strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tensile strength</td>
<td>Fatigue strength</td>
<td></td>
</tr>
</tbody>
</table>

Note: for smooth and polished specimen.

Ex: fatigue ratio ~ 0.5 for cast and wrought steels, ~ 0.35 for non-ferrous.
Fatigue strength improvement by controlling metallurgical variables

By increasing tensile strength

By strengthening mechanisms

- Grain boundary strengthening
- Fibre strengthening
- Second phase strengthening
- Cold working

**Note:** not for all cases and not proportionally.

- Grain size has its greatest effect on fatigue life in the low-stress, high cycle regime.
Fatigue strength improvement by controlling metallurgical variables

By controlling microstructure

- Promote homogeneous slip / plastic deformation through thermomechanical processing → reduces residual stress/ stress concentration.

- Heat treatments to give hardened surface but should avoid stress concentration.

- Avoid inclusions → stress concentration → fatigue strength

- Interstitial atoms increase yield strength, if plus strain aging → fatigue strength

Table 12-4 Influence of inclusions on fatigue limit of SAE 4340 steel

<table>
<thead>
<tr>
<th>Property</th>
<th>Electric-furnace-melted</th>
<th>Vacuum-melted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal fatigue limit, MPa</td>
<td>800</td>
<td>958</td>
</tr>
<tr>
<td>Transverse fatigue limit, MPa</td>
<td>545</td>
<td>827</td>
</tr>
<tr>
<td>Ratio transverse/longitudinal</td>
<td>0.68</td>
<td>0.86</td>
</tr>
<tr>
<td>Hardness, Hc</td>
<td>27</td>
<td>29</td>
</tr>
</tbody>
</table>

Effect of interstitial atoms

Suranaree University of Technology

Tapany Udomphol

May-Aug 2007
Effect of corrosion on fatigue

- **Fatigue corrosion** occurs when material is subjected to cyclic stress in a corrosive condition.

- **Corrosive attack** produces pitting on metal surface. **Pits** act as notches → fatigue strength.

- Chemical attack greatly **accelerates** the rate of fatigue crack propagation.
Corrosion fatigue test can be carried out similar to fatigue test but in a controlled corrosive environment.

- Since corrosion process is a time-dependent phenomenon, the higher the testing speed (frequency), the smaller the damage due to corrosion.

- The action of the cyclic stress causes localised breakdown of the surface oxide film \( \rightarrow \) corrosion pits.
Minimization of corrosion fatigue

- **Select corrosion-resistant materials** for the desired application. 
  *Ex:* stainless steel, bronze, would give better service than heat-treated steel.

- Protection of the metal from contact with the corrosive environment by **protective metallic or non-metallic coatings**.

- **Introducing compressive residual stresses** by nitriding, shot peening → eliminating surface defects.
Effect of temperature on fatigue

- **Temperature** (Increasing $\sigma_{TS}$)
- **Fatigue strength**

If testing temp $< RT \rightarrow$ low temperature fatigue.
If testing temp $> RT \rightarrow$ high temperature fatigue.

- In high temperature fatigue, there is a transition from **fatigue failure** to **creep failure** as the temperature increases (creep dominates at high temperatures).

- **Coarse grained metal** has higher fatigue strength – where creep dominates.

- **Fine grained metal** has higher fatigue strength at low temperatures.

- **Fatigue failure**
  - Transcrystalline fatigue failure

- **Creep failure**
  - Intercrystalline creep failure
Thermal fatigue occurs when metal is subjected to high and low temperature, producing fluctuating cyclic thermal stress.

**Thermal cycle**

- **Cold**
- **Hot**

**Volume change**

- Normally occurs in high temperature equipment.
- Low thermal conductivity and high thermal expansion properties are critical.

- The *thermal stress* developed by a temperature change $\Delta T$ is

$$\sigma = \alpha E \Delta T \quad \text{Eq. 19}$$

Where $\alpha$ is linear thermal coefficient of expansion

$E$ is elastic modulus

If failure occurs by one application of thermal stress, the condition is called *thermal shock*.
There are several distinct philosophies concerning for design for fatigue

1) **Infinite-life design**: Keeping the stress at some fraction of the fatigue limit of the material.

2) **Safe-life design**: Based on the assumption that the material has flaws and has finite life. Safety factor is used to compensate for environmental effects, varieties in material production/manufacturing.

3) **Fail-safe design**: The fatigue cracks will be detected and repaired before it actually causes failure. For aircraft industry.

4) **Damage tolerant design**: Use fracture mechanics to determine whether the existing crack will grow large enough to cause failure.
References

• Lecture note, MRes 2000, School of Metallurgy and Materials, Birmingham University, UK