**Bond Stresses in Beam**

Concrete

Reinforcing bar

End slip

Greased or lubricated

Free slip $\varepsilon_s \neq \varepsilon_c$
Bond forces acting on concrete

Bond forces acting on steel

Tied-arch action in a beam with little or no bond (Round bar reinforcing)
Bond Stress Based on Simple Cracked Analysis

\[ \Sigma M_o = 0 \]
\[ j \, d \, (dT) = V \, (dx) \]
\[ \frac{dT}{dx} = \frac{V}{jd} \]  

\[ \Sigma F_x = 0 \]
\[ u \, \Sigma_o \, dx = dT \]

**Elastic Cracked Section Equation**

\[ u = \frac{V}{\Sigma_o \, j \, d} \]

**ระยำฟังและระยำทำบยองเหล็กเสริม**

แรงดึงในเหล็ก  = \( A_s \, f_s = \pi \, d^2 \, f_s \)

\( d \)

แรงยืดหนี้วิ่ง  = \( \pi \, d \, Lu \)

\( d \)

\( Lu \)

\( \pi \, d \, Lu = \pi \, d^2 \, f_s \)

**ระยะฟังหรือระยะทำบยองเหล็กเสริม**

\[ L = \frac{d \, f_s}{4 \, u} \]
หน่วยแรงยึดเหนี่ยวที่ยอมให้ (WSD)

แหล่งข้อมูลข้อจำแนกแรงตึง:

แหล่งข้อมูล  

แหล่งข้อมูล  

แหล่งข้อมูล  

แหล่งข้อมูล  

* สำหรับแหล่งข้อมูลสมบัติที่ใช้กำลังครึ่งหนึ่งแต่ไม่เกิน 11 กก./ชม. ²

* แหล่งข้อมูลหมายถึงแหล่งข้อมูลที่มีคุณภูมิอยู่ได้แหล่งข้อมูล 30 ชม.

Nominal Bond Strength

แหล่งข้อมูลข้อจำแนกแรงตึง:

แหล่งข้อมูล  

แหล่งข้อมูล  

แหล่งข้อมูล  

แหล่งข้อมูล  

* สำหรับแหล่งข้อมูลสมบัติที่ใช้กำลังครึ่งหนึ่งแต่ไม่เกิน 17.6 กก./ชม. ²
Steel Force and Bond Stress

\[ T = \frac{M}{j_d} \]

\[ u = \frac{1}{\Sigma_o} \frac{dT}{dx} \]

Actual Distribution of Flexural Bond Stress

\[ T = \frac{M}{j_d} \]

\[ u = \frac{V}{\Sigma_o j_d} \]
Mechanism of Bond Transfer

Friction & chemical adhesion between concrete and steel

Reaction of concrete on ribs

Forces exerted by ribs on concrete

Failure modes

Side-split failure

V-notch failure

Vertical cracking of bottom cover
Ultimate Bond Strength

Cylindrical zones of circumferential tension

Reinforcement

Radial component of bearing pressure

Circumferential tensile stresses

Minimum Bar Covering and Spacing

Minimum bar covering ($c_b$)

Minimum bar spacing ($2c_s$)
Splitting of concrete along reinforcement

Embedment length to develop full tensile strength of bar

\[ T = 0 \]

Requirement: \( l \geq l_d \): development length

**Factors influencing \( l_d \)**

- Tensile strength of concrete \( f_{ct} \)
- Steel covering
- Bar spacing
- Stirrup
Deformation length $L_d$ shall be determined from either 12.2.2 or 12.2.3, but shall not be less than 12 in.

12.2.3 – For deformed bars $L_d$ shall be:

$$L_d = \frac{0.28 f_y}{\sqrt{f'_c}} \frac{\alpha \beta \gamma \lambda}{(c + K_{tr})/d_b}$$

in which the term $(c + K_{tr})/d_b$ shall not be taken greater than 2.5, and $K_{tr} = \frac{A_{tr} f_{yt}}{105 s n}$

$c$ = the smaller of

(a) distance from center of bar to the nearest concrete surface

(b) half the center-to-center spacing of the bars

$A_{tr}$ = total cross-sectional area of all transverse reinforcement within spacing $s$

$f_{yt}$ = yield strength of transverse reinforcement

$s$ = maximum spacing of transverse reinforcement within $L_d$

$n$ = number of bars being developed along the plane of splitting.

It shall be permitted to use $K_{tr} = 0$ as a design simplification even if transverse reinforcement is present.

---

**Definition of $A_{tr}$**

**ACI 12.2.5 – Excess reinforcement**

Reduction in $L_d$ shall be permitted where reinforcement in a flexural member is in excess of that required by analysis

$$(A_s \text{ required})/(A_s \text{ provided}) L_d$$

Reduction for large cover and wide bar spacing: 0.8

Heavily confined reinforcement: 0.75
Modifying Multipliers of Development Length in Tension

α  Bar location:  
(a) Top reinforcement  \( \alpha = 1.3 \)
(b) Bottom bars  \( \alpha = 1.0 \)

β  Coating factor:  
(a) Epoxy-coated cover < 3\(d_b\) or clear spacing < 6\(d_b\)  \( \beta = 1.5 \)
(b) All other epoxy-coated  \( \beta = 1.2 \)
(c) Uncoated reinforcement  \( \beta = 1.0 \)

\textit{Note:}  \( \alpha \times \beta \) need not be taken > 1.7

γ  Bar-size factor:  
(a) DB20 and smaller  \( \gamma = 0.8 \)
(b) DB25 and larger  \( \gamma = 1.0 \)

λ  Lightweight aggregate factor:  
(a) Lightweight aggregate concrete  \( \lambda = 1.3 \)
(b) Normal weight concrete  \( \lambda = 1.0 \)

ACI 12.2.2 – Simplified Equation for Development Length

For deformed bars \( L_d \) shall be as follows:

set  \( \frac{c + K_{tr}}{d_b} = 1.5 \)

<table>
<thead>
<tr>
<th>Case A:</th>
<th>DB20 and smaller  ( (\gamma = 0.8) )</th>
<th>DB25 and larger  ( (\gamma = 1.0) )</th>
</tr>
</thead>
</table>
| (1) covering = \( d_b \)  
  clear c-c = \( d_b \)  
  min. stirrup  
| \( \frac{l_d}{d_b} = \frac{0.15 f_y}{\sqrt{f'_c}} \alpha \beta \lambda \)  
  (A - 1) | \( \frac{l_d}{d_b} = \frac{0.19 f_y}{\sqrt{f'_c}} \alpha \beta \lambda \)  
  (A - 2) |
| (2) covering = \( d_b \)  
  clear c-c = 2\(d_b\) | |

Case B: others

<table>
<thead>
<tr>
<th>(B - 1)</th>
<th>(B - 2)</th>
</tr>
</thead>
</table>
| \( \frac{l_d}{d_b} = \frac{0.23 f_y}{\sqrt{f'_c}} \alpha \beta \lambda \)  
  (B - 1) | \( \frac{l_d}{d_b} = \frac{0.28 f_y}{\sqrt{f'_c}} \alpha \beta \lambda \)  
  (B - 2) |
Development Length of Bars in Compression

\[
l_{db} = \frac{0.075 d_b f_y}{\sqrt{f'_c}} \geq 0.0043 d_b f_y
\]

Modification factors:

- Excess reinforcement \( A_s \text{ req'd} / A_s \text{ prv'd} \)
- Spiral stirrup
  \[ \text{dia.} \geq 6 \text{ mm, pitch} \leq 10 \text{ cm} \] 0.75
- Tied stirrup
  DB12, spacing \( \leq 10 \text{ cm} \) 0.75

Development Length for Bundled Bars

when space for proper clearance is restricted

- No more than 4 bars bundled in contact
- Bar larger than DB36 shall not be bundled in beams
- Termination of each bar must be different at least 40\(d_b\)
- Development length based on single bar in the bundle
  - Increase 20% for 3 bars
  - Increase 33% for 4 bars
Anchorage of Tension Bars by Hooks

When straight embedment is not enough

a) Standard Bar Hook

180° Hook

\[ T \]

\[ d_b \]

\[ D = \text{diam of bend} \]

\[ 4d_b \geq 6 \text{ cm} \]

90° Hook

\[ T \]

\[ d_b \]

\[ D \]

\[ 12d_b \]

For stirrup and tie anchorage only:

(a) For DB16 or smaller

(b) For DB20 or DB25

(c) For DB25 or smaller

Min. Dia. of Bend (D)

<table>
<thead>
<tr>
<th></th>
<th>DB10 - DB25</th>
<th>DB28 - DB36</th>
<th>DB40 - DB60</th>
</tr>
</thead>
<tbody>
<tr>
<td>DB10 or smaller</td>
<td>6d_b</td>
<td>8d_b</td>
<td>10d_b</td>
</tr>
</tbody>
</table>

For DB16 or smaller

\[ T \]

\[ d_b \]

\[ D \]

\[ 6d_b \]

For DB20 or DB25

\[ T \]

\[ d_b \]

\[ D \]

\[ 12d_b \]

For DB25 or smaller

\[ T \]

\[ d_b \]

\[ 135° \]

\[ 6d_b \]
**b) Development length and Hook**

Combined actions:
- Bond along straight length
- Anchorage provided by hook

![Diagram of development length and hook](image)

- Critical section
  - Full bar tension

Basic development length with $f_y = 4,000$ kg/cm²

$$L_{hb} = \frac{318d_b}{\sqrt{f_c'}}$$

$$L_{dh} = \text{Modification factor} \times L_{hb}$$

**Modification factors:**

- Bar with $f_y$ other than 4,000 ksc
  - $f_y / 4,000$

- DB25mm and smaller
  - Side cover not less than 4 cm
  - 90° hook cover hook not less than 5 cm
  - 0.7

- DB25mm and smaller
  - Hook enclosed within stirrup along $l_{dh}$
  - Stirrup spacing not greater than $3d_b$
  - 0.8

- Excess reinforcement
  - $A_s$ reqd / $A_s$ prvd

- Lightweight concrete
  - 1.3

- Epoxy coating
  - 1.2
EXAMPLE 11-1 Anchorage of a Straight Bar

A 40-cm-wide cantilever beam frames into the edge of a 40-cm-thick wall, as shown. At ultimate, the DB25 bars at the top of the cantilever are stressed to their yield strength at point \( A \) at the face of the wall. Compute the minimum embedment of the bars into the wall. The concrete is normal-weight concrete with a strength of 240 ksc. The yield strength of the flexural reinforcement is 4,000 ksc.

1. Find the spacing and confinement case.

There are no stirrups or ties in the wall, but there are DB16 at 30 cm o.c. vertical bars outside of the 3DB25

- Clear side cover = \( 4 + 1.6 = 5.6 \text{ cm} \) (2.24\( d_b \))
- Clear spacing = \( (40 – 2(4+1.6) – 3\times 2.5) / 2 = 10.65 \text{ cm} \) (4.26\( d_b \))
- สำหรับเหล็กกัน\( \alpha = 1.3 \)
- เหล็กไม่เหลี่ยม\( \beta = 1.0 \)
- สำหรับคอนกรีต\( \lambda = 1.0 \)

Since the clear spacing between the bars is not less than 2\( d_b \) and the clear cover exceeds \( d_b \), case (A-2) applies.

2. Compute the development length. From (A-2):

\[
L_d = \frac{0.19 f_y \alpha \beta \lambda}{f'_c} d_b = \frac{0.19(4,000)(1.3)(1.0)(1.0)}{\sqrt{240}} \times 2.5 = 159.4 \text{ cm}
\]

Use development length \( L_d = 160 \text{ cm} \)
3. Compute the development length. Detailed equation:

\[ L_d = \frac{0.28 f_y \alpha \beta \gamma \lambda}{\sqrt{f'_c \left( \frac{c + K_{tr}}{d_b} \right)}} d_b \]

\( c = \) the smaller of

(a) distance from center of bar to the nearest concrete surface:

the side cover is \( 4 + 1.6 + 2.5/2 = 6.85 \) cm;

(b) half the center-to-center spacing of the bars, or

\[ 0.5 \left( \frac{40 - 2 \times 6.85}{2} \right) = 6.58 \text{ cm} \]

Therefore, \( c = 6.58 \text{ cm} \).

\[ K_{tr} = \frac{A_{tr} f_{yt}}{105 s n} = \frac{4.02 \times 4,000}{105 \times 30 \times 3} = 1.7 \text{ cm} \]

\( s = \) spacing of the transverse reinforcement within \( L_d = 30 \text{ cm} \)

\( A_{tr} = \) transverse steel area crossing splitting plane within spacing \( s \)

= DB16 at 30 cm each face = \( 2 \times 2.01 = 4.02 \text{ cm}^2 \)

\( f_{yt} = 4,000 \text{ kg/cm}^2 \) for the wall steel

\( n = \) number of bars being anchored = \( 3 \)

\[ \frac{c + K_{tr}}{d_b} = \frac{6.58 + 1.7}{2.5} = 3.31 > 2.5 \]

\[ L_d = \frac{0.28 f_y \alpha \beta \gamma \lambda}{\sqrt{f'_c \left( \frac{c + K_{tr}}{d_b} \right)}} d_b = \frac{0.28 \times 4,000 \times 1.3 \times 1.0 \times 1.3 \times 1.0}{\sqrt{240 \times 2.5}} \times 2.5 \]

\[ = 122.2 \text{ cm} \quad \text{Use development length } L_d = 125 \text{ cm} \]
Bar Cutoff and Bend Points in Beams

Moment capacity of beam:

\[ M = A_s f_y \left( d - \frac{a}{2} \right) \]

Bar Cutoff requirements of the ACI Code

Face of support

Moment capacity of bars \( M \)

Inflection point for \( +A_s \)

Inflection point for \( -A_s \)

Greatest of \( d, 12d_b \) or \( L_n/16 \) for at least \( 1/3 \) of \( -A_s \)

Moment capacity of bars \( O \)

Bars \( M \)

Bars \( L \)

Bars \( N \)

Bars \( O \)

15 cm for at least \( 1/4 \) of \( +A_s \)

\( L_d \)
Development of Reinforcement at Simple Supports

At point A, \( M_u = \phi M_n \) of bars continuing into the support

ACI: Distance from A to bar end \( \geq L_d \)

Approximate

\( V_{avg} \) (say, 0.9\( V_u \)) \( x = \phi M_n \) → \( x = \frac{M_n}{V_u} \)

\( L_a = \) Straight length beyond c.g. support

ACI: Embedment length

\[
L_a + 1.3 \frac{M_n}{V_u} \geq L_d
\]

1.3 for less tendency to split due to confinement at support

Development of Reinforcement at Inflection Points

Locate away from support → no confinement → 1.3 factor not apply

ACI: Embedment length

Usable \( L_a + \frac{M_n}{V_u} \geq L_d \)

Usable \( L_a = \) Actual \( L_a \) but not exceed the larger of 12\( d_b \) or \( d \)
Standard Cutoff and Bend Points for Bars

For approximately equal spans with uniformly distributed loads

<table>
<thead>
<tr>
<th>L₁/4</th>
<th>L₁/3</th>
<th>L₂/3</th>
<th>L₂/3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0 cm</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

wₜₜ = 8.0
f'ₙ = 280 กก./ซม.², f_y = 4,000 กก./ซม.²,
b = 40 ซม., h = 60 ซม. และคอนกรีตที่ 4 ซม.

Exterior column

Interior column

Lₙ = 7.6 m

1. ออกแบบเหล็กเสริมรับบ่อน้ำหนักและแรงเฉือนเบื้องต้น

   a. ใช้วิเคราะห์แบบประมาณตำแหน่งและแรงเฉือน

<p>| Interior face of exterior support | -M_u = w_uL_n²/16 = 8(7.6)²/16 = -28.88 t-m |</p>
<table>
<thead>
<tr>
<th>Condition</th>
<th>Equation</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mid span positive</td>
<td>$+M_u = w_u L_n^2/14 = 8(7.6)^2/14 = 33.01 \text{ t-m}$</td>
<td></td>
</tr>
<tr>
<td>Exterior face of first interior support</td>
<td>$-M_u = w_u L_n^2/10 = 8(7.6)^2/10 = -46.21 \text{ t-m}$</td>
<td></td>
</tr>
<tr>
<td>Exterior face of first interior support</td>
<td>$V_u = 1.15 w_u L_n/2 = 1.15(8)(7.6)/2 = 34.96 \text{ t-m}$</td>
<td></td>
</tr>
</tbody>
</table>

b. พิจารณาเหล็กเสริมรับแรงต้านดัด โดยใช้คอนกรีตที่มี $4 \text{ cm}$. เหล็กปลอก DB10 และเหล็กเสริมรับแรงต้าน DB25 หรือ DB28 ค่า $d \approx 60-4-1.0-1.4 \approx 53.6 \text{ cm}$.

<table>
<thead>
<tr>
<th>$M_u$</th>
<th>$A_s$ required</th>
<th>Bars</th>
<th>$A_s$ provided</th>
</tr>
</thead>
<tbody>
<tr>
<td>-28.88</td>
<td>15.97 cm$^2$</td>
<td>4DB25</td>
<td>19.63 cm$^2$</td>
</tr>
<tr>
<td>+33.01</td>
<td>18.44 cm$^2$</td>
<td>4DB25</td>
<td>19.63 cm$^2$</td>
</tr>
<tr>
<td>-46.21</td>
<td>26.76 cm$^2$</td>
<td>2DB25+3DB28</td>
<td>28.29 cm$^2$</td>
</tr>
</tbody>
</table>

![Diagram](image-url)