APPROXIMATE ANALYSIS OF STATICALLY INDETERMINATE STRUCTURES

- Trusses
- Vertical Loads on Building Frames
- Lateral Loads on Building Frames: Portal Method
- Lateral Loads on Building Frames: Cantilever Method Problems
Method 1: If the diagonals are intentionally designed to be long and slender, it is reasonable to assume that the panel shear is resisted entirely by the tension diagonal, whereas the compressive diagonal is assumed to be a zero-force member.

Method 2: If the diagonal members are intended to be constructed from large rolled sections such as angles or channels, we will assume that the tension and compression diagonals each carry half the panel shear.
Example 1

Determine (approximately) the forces in the members of the truss shown in Figure. The diagonals are to be designed to support both tensile and compressive forces, and therefore each is assumed to carry half the panel shear. The support reactions have been computed.
\[ \sum F_y = 0: \quad 20 - 10 - 2F \sin(36.87^\circ) = 0 \]

\[ F = 8.33 \text{ kN} \]

\[ F_{FB} = 8.33 \text{ kN (T)} \]

\[ F_{AE} = 8.33 \text{ kN (C)} \]

\[ \sum M_A = 0: \quad F_{FE}(3) - 8.33 \cos(36.87^\circ)(3) = 0 \]

\[ F_{FE} = 6.67 \text{ kN (C)} \]

\[ \sum M_F = 0: \quad F_{AB}(3) - 8.33 \cos(36.87^\circ)(3) = 0 \]

\[ F_{AB} = 6.67 \text{ kN (T)} \]

\[ \sum F_y = 0: \quad F_{AF} - 10 - 8.33 \sin(36.87^\circ) = 0 \]

\[ F_{AF} = 15 \text{ kN (T)} \]
\[ \theta = 36.87^\circ \]

\[ V = 10 \text{ kN} \]

\[ F = 8.33 \text{ kN} \]

\[ F_{DB} = 8.33 \text{ kN (T)} \]

\[ F_{EC} = 8.33 \text{ kN (C)} \]

\[ \sum F_y = 0: \quad 10 - 2F \sin(36.87^\circ) = 0 \]

\[ F = 8.33 \text{ kN} \]

\[ F_{DC} = 5 \text{ kN (C)} \]

\[ \sum M_C = 0: \quad F_{ED}(3) - 8.33 \cos 36.87^\circ (3) = 0 \]

\[ F_{ED} = 6.67 \text{ kN (C)} \]

\[ \sum M_D = 0: \quad F_{BC}(3) - 8.33 \cos 36.87^\circ (3) = 0 \]

\[ F_{BC} = 6.67 \text{ kN (T)} \]

\[ F_{EB} = 2(8.33 \sin 36.87^\circ) = 10 \text{ kN (T)} \]
Example 2

Determine (approximately) the forces in the members of the truss shown in Figure. The diagonals are slender and therefore will not support a compressive force. The support reactions have been computed.
\[ F_{AI} = 0 \]

\[ \Sigma F_y = 0: \quad 40 - 10 - F_{JB}\cos 45^\circ = 0 \]
\[ F_{JB} = 42.43 \text{ kN (T)} \]

\[ \Sigma M_A = 0: \quad F_{JI}(4) - 42.43\sin 45^\circ(4) = 0 \]
\[ F_{JI} = 30 \text{ kN (C)} \]

\[ \Sigma M_J = 0: \quad F_{AB}(4) = 0 \]
\[ F_{AB} = 0 \]

\[ \Sigma F_y = 0: \quad 40 - 10 - F_{JB}\cos 45^\circ = 0 \]
\[ F_{JB} = 42.43 \text{ kN (T)} \]

\[ \Sigma M_A = 0: \quad F_{JI}(4) - 42.43\sin 45^\circ(4) = 0 \]
\[ F_{JI} = 30 \text{ kN (C)} \]

\[ \Sigma M_J = 0: \quad F_{AB}(4) = 0 \]
\[ F_{AB} = 0 \]
\[ \Sigma M_B = 0: \]
\[ F_{IH}(4) - 14.14\sin 45^\circ(4) + 10(4) - 40(4) = 0 \]
\[ F_{JH} = 40 \text{ kN (C)} \]

\[ \Sigma M_I = 0: \]
\[ F_{BC}(4) - 40(4) + 10(4) = 0 \]
\[ F_{BC} = 30 \text{ kN (T)} \]

\[ F_{BI} = 2(14.14\times 42.3 \sin 45^\circ) = 20 \text{ kN (C)} \]
Vertical Loads on Building Frames

typical building frame
• Assumptions for Approximate Analysis

(a) Simply supported column with a girder:

- Point of zero moment at point A:
  - Assumed points of zero moment:
    - Point A: 0.1L
    - Point B: 0.21L

(b) Diagram showing the point of zero moment and the approximate case:

(c) Simply supported model with points of zero moment:

- Point A: 0.1L
- Point B: 0.21L
- Point C: 0.8L
- Point D: 0.1L
Example 3

Determine (approximately) the moment at the joints $E$ and $C$ caused by members $EF$ and $CD$ of the building bent in the figure.
\[ 0.6(0.3) + 2.4(0.6) = 1.62 \text{kN} \cdot \text{m} \]

\[ 0.6 \times 2.4 + 4.8 \times 2.4 = 1.62 \text{kN} \cdot \text{m} \]
Portal Frames and Trusses

- Frames: Pin-Supported

![Diagram of Portal Frames and Trusses with labeled forces and moments](image)

(a) Portal Frame with forces and moments labeled.

(b) Portal Frame with assumed hinge and displacement labeled.

(c) Portal Frame with forces and moments labeled.

(d) Diagram showing internal forces in a portal frame.
• Frames: Fixed-Supported

(a)

(b)

(c)

(d)
• Frames: Partial Fixity

- (a)

- (b)

• Trusses

- (a)

- (b)
Example 4

Determine by approximate methods the forces acting in the members of the Warren portal shown in the figure.
$\frac{40}{2} = 20 \text{kN} = V$

$20 \text{kN} = V$

$V = 20 \text{kN}$

$40 \text{kN}$

$3.5 \text{m}$

$2 \text{m}$

$2 \text{m}$

$B$

$H$

$G$

$J$

$K$

$C$

$D$

$E$

$F$

$N$

$N$

$N$

$N$

$V = 20 \text{kN}$

$V = 20 \text{kN}$

$\Sigma M_J = 0$:

$N(8) - 40(5.5) = 0$

$N = 27.5 \text{kN}$

$\Sigma M_A = 0$:

$M - 20(3.5) = 0$

$M = 70 \text{kN} \cdot \text{m}$
\[ \Sigma F_y = 0: \quad -27.5 + F_{BD} \cos 45^\circ = 0 \]
\[ F_{BD} = 38.9 \text{ kN (T)} \]

\[ \Sigma M_B = 0: \quad F_{CD}(2) - 40(2) - 20(3.5) = 0 \]
\[ F_{CD} = 75 \text{ kN (C)} \]

\[ \Sigma M_D = 0: \quad F_{BH}(2) + 27.5(2) - 20(5.5) = 0 \]
\[ F_{BH} = 27.5 \text{ kN (T)} \]

\[ \Sigma F_y = 0: \quad 27.5 - F_{EG} \cos 45^\circ = 0 \]
\[ F_{EG} = 38.9 \text{ kN (C)} \]

\[ \Sigma M_G = 0: \quad F_{EF}(2) - 20(3.5) = 0 \]
\[ F_{EF} = 35 \text{ kN (T)} \]

\[ \Sigma M_E = 0: \quad F_{GH}(2) + 27.5(2) - 20(5.5) = 0 \]
\[ F_{GH} = 27.5 \text{ kN (C)} \]
\[ \Sigma F_y = 0: \quad F_{DH} \sin 45^\circ - 38.9 \sin 45^\circ = 0 \]

\[ F_{DH} = 38.9 \text{ kN (C)} \]

\[ \Sigma F_x = 0: \quad 75 - 2(38.9 \cos 45^\circ) - F_{DE} = 0 \]

\[ F_{DE} = 20 \text{ kN (C)} \]

\[ \Sigma F_y = 0: \quad F_{HE} \sin 45^\circ - 38.9 \sin 45^\circ = 0 \]

\[ F_{HE} = 38.9 \text{ kN (T)} \]
Lateral Loads on Building Frames: Portal Method

\[ \circ \text{ = inflection point} \]

(a)

\[ \circ \text{ = inflection point} \]

(b)
Example 5

Determine by approximate methods the forces acting in the members of the Warren portal shown in the figure.
\[\Sigma F_x = 0: \quad 5 - 6V = 0\]

\[V = 0.833 \text{ kN}\]
Example 6

Determine (approximately) the reactions at the base of the columns of the frame shown in Fig. 7-14a. Use the portal method of analysis.
$$\Sigma F_x = 0: \quad 20 - 4V = 0 \quad V = 5 \text{ kN}$$

$$\Sigma F_x = 0: \quad 20 + 30 - 4V' = 0 \quad V' = 12.5 \text{ kN}$$
In summary, using the cantilever method, the following assumptions apply to a fixed-supported frame.

1. A hinge is place at the center of each girder, since this is assumed to be point of zero moment.

2. A hinge is placed at the center of each column, since this is assumed to be a point of zero moment.

3. The axial stress in a column is proportional to its distance from the centroid of the cross-sectional areas of the columns at a given floor level. Since stress equals force per area, then in the special case of the columns having equal cross-sectional areas, the force in a column is also proportional to its distance from the centroid of the column areas.
**Example 7**

Determine (approximately) the reactions at the base of the columns of the frame shown. The columns are assumed to have equal crosssectional areas. Use the cantilever method of analysis.
\[
\bar{x} = \frac{\sum \tilde{x}A}{\sum A} = \frac{0(A) + 6(A)}{A + A} = 3
\]
\[ \Sigma M_M = 0: \quad -30(2) + 3H_y + 3K_y = 0 \]

The unknowns can be related by proportional triangles, that is
\[ \frac{H_y}{3} = \frac{K_y}{3} \quad \text{or} \quad H_y = K_y \]

\[ H_y = K_y = 10 \text{kN} \]

\[ \Sigma M_N = 0: \quad -30(6) - 15(2) + 3G_y + 3L_y = 0 \]

The unknowns can be related by proportional triangles, that is
\[ \frac{G_y}{3} = \frac{G_y}{3} \quad \text{or} \quad G_y = L_y \]

\[ G_y = L_y = 35 \text{kN} \]
Example 8

Show how to determine (approximately) the reactions at the base of the columns of the frame shown. The columns have the cross-sectional areas shown. Use the cantilever of analysis.
\[
\bar{x} = \frac{\sum \tilde{x}A}{\sum A} = \frac{6000(0) + 5000(6) + 4000(10) + 6000(18)}{6000 + 5000 + 4000 + 6000} = 8.48 \text{ m}
\]
\( \Sigma M_{NA} = 0: \quad -35(2) + L_y(8.48) + M_y(2.48) + N_y(1.52) + O_y(9.52) = 0 \quad -----(1) \)

Since any column stress \( \sigma \) is proportional to its distance from the neutral axis

\[
\sigma_M = \frac{2.48}{8.48} \sigma_L; \quad \sigma_N = \frac{1.52}{8.48} \sigma_L; \quad \sigma_O = \frac{9.52}{8.48} \sigma_L; \quad \frac{M_y}{5000(10^{-6})} = \frac{2.48}{8.48} \left( \frac{L_y}{6000(10^{-6})} \right) \quad -----(2)
\]

\[
\frac{N_y}{4000(10^{-6})} = \frac{1.52}{8.48} \left( \frac{L_y}{6000(10^{-6})} \right) \quad -----(3)
\]

\[
\frac{O_y}{6000(10^{-6})} = \frac{9.52}{8.48} \left( \frac{L_y}{6000(10^{-6})} \right) \quad -----(4)
\]

Solving Eqs. (1) - (4) yields

\( L_y = 3.508 \text{ kN} \)

\( M_y = 0.855 \text{ kN} \)

\( N_y = 0.419 \text{ kN} \)

\( O_y = 3.938 \text{ kN} \)
\[ + \nabla \Sigma M_{NA} = 0: \quad -45(3) - 35(7) + E_y(8.48) + F_y(2.48) + G_y(1.52) + H_y(9.52) = 0 \quad ----- (5) \]

Since any column stress \( \sigma \) is proportional to its distance from the neutral axis:

\[
\frac{\sigma_F}{2.48} = \frac{\sigma_E}{8.48}; \quad \sigma_F = \frac{2.48}{8.48} \sigma_E; \quad \frac{F_y}{5000(10^{-6})} = \frac{2.48}{8.48} \left( \frac{E_y}{6000(10^{-6})} \right) \quad ----- (6)
\]

\[
\frac{\sigma_G}{1.52} = \frac{\sigma_E}{8.48}; \quad \sigma_G = \frac{1.52}{8.48} \sigma_E; \quad \frac{G_y}{4000(10^{-6})} = \frac{1.52}{8.48} \left( \frac{E_y}{6000(10^{-6})} \right) \quad ----- (7)
\]

\[
\frac{\sigma_H}{9.52} = \frac{\sigma_E}{8.48}; \quad \sigma_H = \frac{9.52}{8.48} \sigma_E; \quad \frac{H_y}{6000(10^{-6})} = \frac{9.52}{8.48} \left( \frac{E_y}{6000(10^{-6})} \right) \quad ----- (8)
\]

Solving Eqs. (1) - (4) yields \( E_y = 19.044 \text{ kN} \quad F_y = 4.641 \text{ kN} \quad G_y = 2.276 \text{ kN} \quad H_y = 21.38 \text{ kN} \)
One can continue to analyze the other segments in sequence, i.e., PQM, then MJFI, then FB, and so on.